Auroras - Why do they occur where they do?

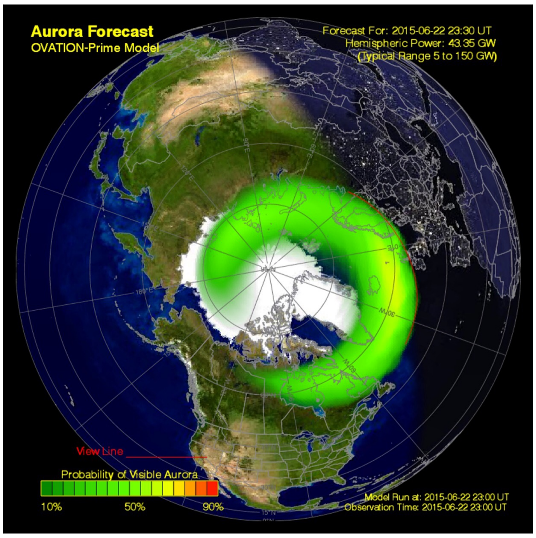
A study of the movement of charged particles in magnetic fields

Introduction

From your former work with auroras and magnetic field tasks you already know:

- that auroras are due to electrically charged particles from the Sun interacting with ions nd other particles in the Earth's atmosphere.
- that the Earth got a magnetic field and what the field lines look like.
- that moving charged particles themselves induce magnetfic fields.
- that auroras can be detected by observing desturbances of the Earth's magnetic field.

What you perhaps do not know is why the auroras only occur where they do. When looking at aurora forecasts the auroras are depicted as circles/ovals as shown on figure 1.



Figur 1: Example of an aurora forecast.

Several questions may occur:

- Why are the auroras shaped as circles/ovals?
- Why are the circles not complete?
- Why do the auroras not cover the poles completely?
- Why are there no auroras towards the equator?

To by able to answer these questions you need knowledge of how an electrically charged particle moves in a magnetic field. This is described by **Lorentz's law**.

Lorentz's law

Lorentz's law describes the *force* on an electrically charged particle moving through *both* an electric field and a magnetic field and is given by the following equation:

$$\vec{F} = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B}\,,$$

where $\vec{F}$ is the force on the particle, q is the charge of the particle, $\vec{E}$ is the *external* electric field from *other* electric sources, $\vec{v}$ is the instantaneous velocity of the particle and $\vec{B}$ is the *external* magnetic field acting on the particle. The letters with arrows above them represent *vectors* [1].

In the absent of an external electric field Lorentz's law reduces to

$$\boxed{\vec{F} = q \cdot \vec{v} \times \vec{B}}$$

which describes the force on an electrically charged particle moving only i an magnetic field.

The $\times$-sign is not an ordinary multiplication sign but rather the sign for the *cross product* between two vectors.

Now it is your turn to search for information. If you are not already familiar with vectors (in three dimensions) and crossproducts please consult your favorite textbooks about the subject or search the internet for reliable sources. Some extra terms to search for might be

- the right hand rule
- perpendicular
- parallel
- direction

[1] some textbooks may refer to vectors with **bold** fonts.

- magnitude

Now you should be able to at least

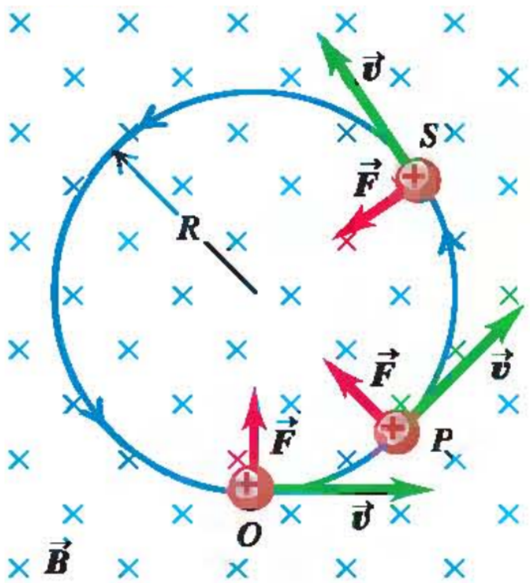
- qualitative tell the direction of the force compared to the velocity and the magnetic field.
- calculate the magnitude of the force when the velocity and the magnetic field are perpendicular to each other.

Motion of a charged particle in a magnetic field

From your earlier search it should clear that

> Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

This is due to the magnetic force *always* being perpendicular to the velocity and hence cannot do *work* on the particle. The magnetic force can not change the *speed* of the particle but only the direction. This phenomenon is shown in figure 2.



Figur 2: A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other. The $\vec{B}$ field points into the paper.

Circular motion

When velocity and magnetic field are perpendicular to each other the particle undergoes circular motion as described in the previous section. When the magnetic force is the only force acting on the particle the motion can be described by Newton's second law and *centripetal* forces[2] resulting in the following equation

$$\boxed{|q| \cdot v \cdot B = m \cdot \frac{v^2}{R}\,,}$$

- *Solve the equation for the radius of the circular path R.*
- *Solve the equation for the speed v.*
- *Solve the equation for the magnetic field strength B.*

Use these equations and your newly aquired knowledge when working with the following exercises. They do not all deal with auroras per se. Some also demonstrate the utilization of magnetic fields to identify and measure different properties of particle in the field of particle physics.

Exercise 1

A proton moves at $7.50 \cdot 10^7 m/s$ perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

Exercise 2

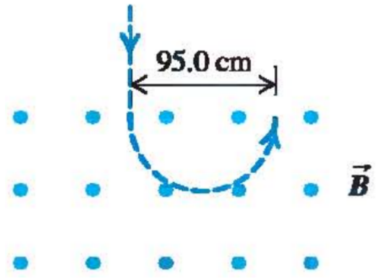
A cosmic ray electron moves at $7.5 \cdot 10^6 m/s$ perpendicular to the Earth's magnetic field at an altitude where field strength is $1.0 \cdot 10^{-5} T$.

- What is the radius of the circular path the electron follows?

[2] *Time to search for information. If not already acquainted with the principle of centripetal forces please look it up.*

Exercise 3

In an experiment with cosmic rays, a vertical beam of particles that have charge of magnitude $3e$ (3 times the *elementary* charge, look it up in a textbook) and mass 12 times the proton mass enters a uniform horizontal magnetic field of 0.250 T and is bent in a semicircle of diameter 95.0 cm, as shown in the figure.

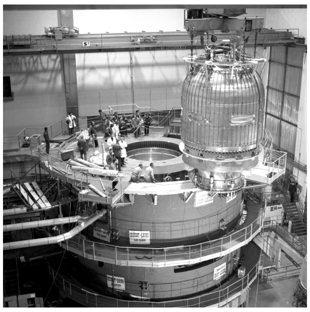
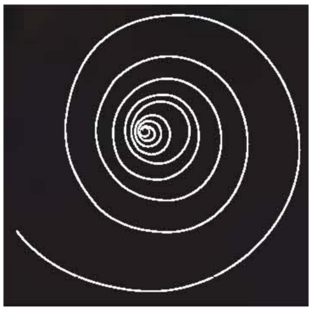


1. Figure out which way the magnetic field points on the pigure.
2. Find the speed of the particles and the sign of their charge.
3. Is it reasonable to ignore the gravity force on the particles?
4. How does the speed of the particles as they enter the field compare to their speed as they exit the field?

Exercise 4

Certain types of bubble chambers (former particle detectors for particle physics. The installation of the BEBC in CERN in the 1970s can be seend on the picture to the left) are filled with liquid hydrogen. When a particle (such as an electron or a proton) passes through the liquid, it leavs a track of bubbles, which can be photographed to show the path of the particle. The detector is immersed in a known magnetic field, which causes the particle to curve. The figure on the right is a trace of a bubble chamber image showing the path of an electron.

1. How could you determine the *sign* of the charge of a particle from a photograph of its path?
2. How can physicists determine the *momentum* (momentum is $\vec{p} = m \cdot \vec{v}$) and the *speed* of this electron by using measurements made on the photograph, given that the magnetic field is known and is perpendicular to the plane of the figure?
3. The electron is obviously spiraling into smaller and smaller circes. What properties of the electron must be changing to cause this behavior? Why does this happen?
4. What would be the path of a neutron in a bubble chamber, and why?

Questions

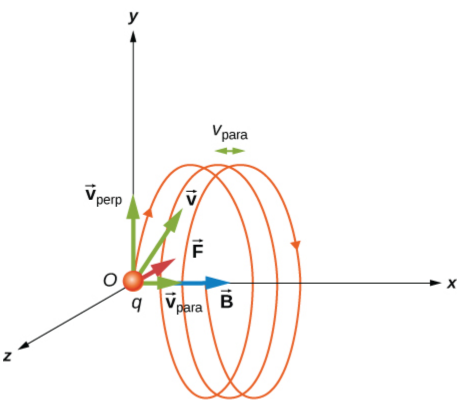
Before concluding this section about circular motion please think about the following two questions.

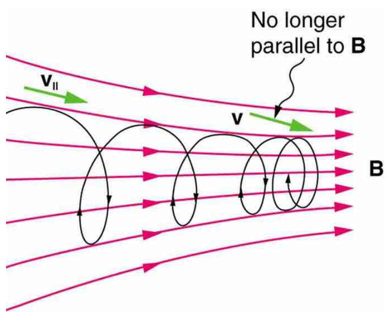
- Can a charged particle move through a magnetic field without experiencing any force? If so, how? If not, why not?
- The magnetic force on a moving charged particle is always perpendicular to the magnetic field $\vec{B}$. Is the trajectory of a moving charged particle always perpendicular to the magnetic field lines? Explain your reasoning.

Helical motion

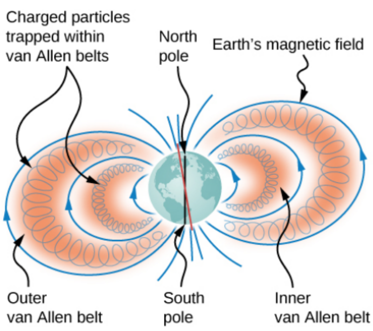
If the velocity of the charged particle is not initially perpendicular to the magnetic field the motion will not be circular. The perpendicular component of the velocity vector, call it v_{perp}, will still produce a magnetic force perpendicular to both this velocity and the magnetic field, while the parallel component, call it v_{para}, will create a constant motion along the *same* direction as the magnetic field. The result is a **helical motion**, as shown in the left figure. The parallel motion determines the *pitch* p of the helix, which is the distande between adjacent turns. This distance is equal to the parallel component of the velocity times the period of the circular part of the motion:

$$p = v_{\text{para}} \cdot T \,.$$





If the magnetic field strength increases in the direction of motion, the magnetic field will exert a force to slow the motion along the field line, forming a kind of magnetic mirror which is able to even reflect the velocity of the particle. This is shown in figure to the right [3]. If the reflection happens at both ends of the magnetic field lines, the particle is trapped in a so-called magnetic bottle. This happens in the *Van Allen* radiation belts around Earth, where charged particles are trapped within the Earth's magnetic field lines as shown in the figure.



[3]The phenomenon is described by Lenz's law. Feel free to look it up.

Can you now explain the shape of the auroras?

Do you remember the questions from the beginning of this document? If not here they are again:

- Why are the auroras shaped as circles/ovals?
- Why are the circles not complete?
- Why do the auroras not cover the poles completely?
- Why are there no auroras towards the equator?

Use your newfound knowledge and perhaps the following figures to answer the questions.