NAROM | ANDØYA SPACE CENTER

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ROCK STAR

ENHANCING STEM EDUCATION

Model Rocket Calculation and Simulation

Summary

In this exercise, students will use technical specifications for the motor and rocket to model how the rocket will behave during flight. In addition, learn more about the stability of rockets and aerodynamic considerations.

Content

Overview	
Purpose	This exercise will provide students with an understanding of how model rocket calculations using basics physics.
Duration	Two hours
Location	Saturn, Rhea and Titan
Equipment	Pen/Pencil Calculator Cardboard with outline of the rocket Model Rocket kit PC
Safety issues	None
Background	Basics physics, familiarity with Newtons law and equations of motion.
Literature	Included
Software	Rocksim: https://www.apogeerockets.com/Rocket_Software/RockSim Alternative open source – OpenRocket: http://openrocket.info/

Learning objectives

Gain increasing knowledge about:

- how physics can be used in practice to calculate model rocket properties
- Get to know important principles related to rocket stability

21st Century skills:

- Work in international groups using English as communication language
- Problem solving skills
- Critical thinking
- Collaboration
- Communication
- Information and media literacy

1 Theory: background

In this exercise, students will use technical specifications for the engine and rocket to model how the rocket will behave in flight. In addition, learn more about the stability of rockets and aerodynamic considerations.

1.1 Rocket Motor

Model rocket engines are solid-state engines that utilize chemical energy to create thrust. Most often the engines use black powder, a combination of potassium nitrate, sulfur and coal as fuel. The fuel is compressed to a solid mass and sealed tightly by the side walls of the rocket engine. A ceramic nozzle accelerates the gas from the engine. When all the fuel in the engine is burned, an exhaust charge causes the parachute to be pushed out in the opposite direction. A cut out of a model rocket is shown in Figure 1.

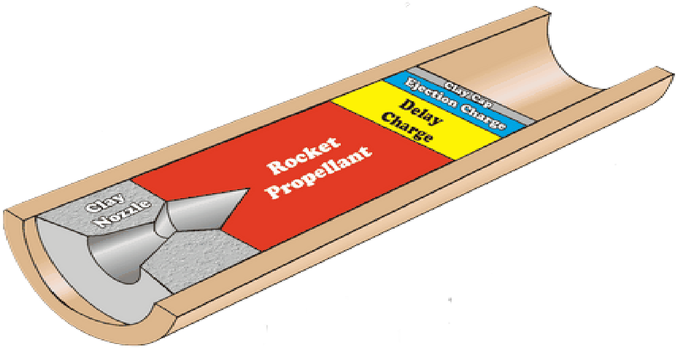


Figure 1. Model Rocket Motor cut-out.

The rocket engines are divided into different categories that provide information about how much thrust they have, how long the burning time they have and whether they can shoot out a parachute and if so when this is going to happen. The categories are usually given as a letter in front of two numbers, e.g. A6-4. The letter tells us how much total impulse (I_t) the rocket engine has with increasing total impulse from A and onwards as shown in Table 1.

Table 1. Total impulse model rockets.

Letter	Total impulse - I_t [Ns]
A	2,5
B	5
C	10
D	20

The first number that stands by the letter indicates the average thrust (F_{ave}). The last number indicates how long it takes in seconds from the rocket engine has stopped burning until it shoots out the parachute. This time is called delay charge (t_d). In the table below is an example for the motor A6-4.

Table 2. Example rocket motor A6-4

A	6	4
Total impulse - I_t [Ns]	Average Thrust - F_{ave} [N]	Delay - t_d [s]
2,5	6	4

The mathematical relation between total impulse (I_t), average thrust (F_{ave}) and the engine's burn time (t_b) is given as formula 1. If one sees the plots of thrust as the function of time in Table 3 the total impulse is the area under the graph of the respective engine category.

Total impulse $$I_t = \int_0^t F_{thrust}(t')dt' = F_{ave}t_b \quad (1)$$

Calculations on how efficient a rocket engine produces thrust can be done by using average fuel consumption q_p, exhaust speed v_e and specific impulse I_{sp} were

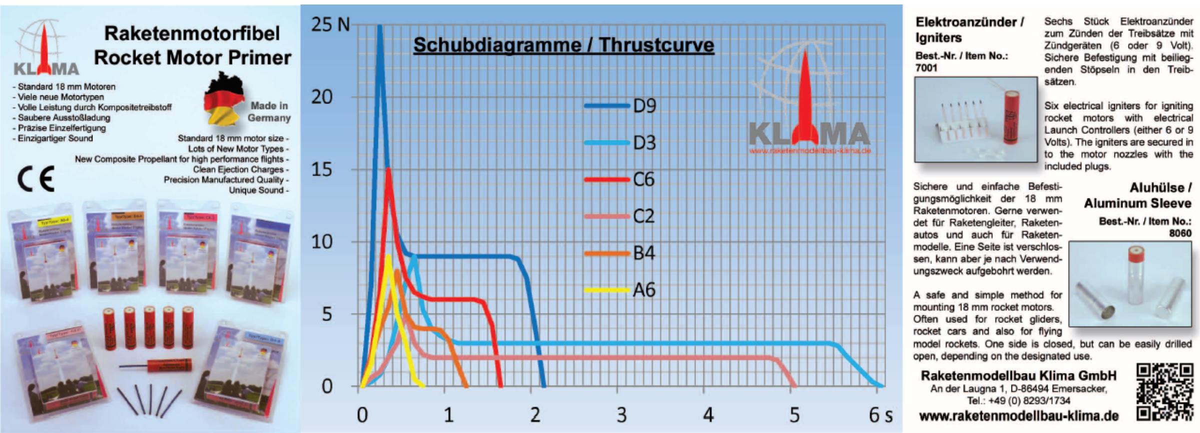
- m_p is the propellant mass
- t_b is the burn time of the motor
- g_0 standard gravity (9,81m/s^2)

Fuel consumption $$q_p = \frac{m_p}{t_b} \quad (2)$$

Exhaust speed $$v_e = \frac{I_t}{m_p} = \frac{F_{ave}}{q_p} \quad (3)$$

Specific impulse $$I_{sp} = \frac{v_e}{g_0} \quad (4)$$

Table 3. Technical specification for Klima rocket motors (Source: Klima GmbH)



Best.-Nr. Item No.	Typ Type	Gesamtimpuls Total Impulse	Schub avg. Thrust	Schubdauer Burning Period	Verzögerung Delay	Gesamtgewicht Total Weight	Pulvergewicht Powder Weight	Max. Startgewicht Max. Lift-Off Wgt.	Verwendung Usage
0164	A6-4	2,5 Ns	6 N	ca. 0,6 s	ca. 4 s	14,5 g	3,5 g	70 g	Single Stage, Final Stage
0240	B4-0	5,0 Ns	4 N	ca. 1,3 s	0 s	15,8 g	5,0 g	90 g	Booster Stage
0244	B4-4	5,0 Ns	4 N	ca. 1,3 s	ca. 4 s	16,8 g	5,8 g	90 g	Single Stage, Final Stage
0320	C2-0	10,0 Ns	2 N	ca. 5,0 s	0 s	22,3 g	11,5 g	150 g	Boost Glider, Rocket Car
0329	C2-P	10,0 Ns	2 N	ca. 5,0 s	verschlossen / plugged	22,5 g	11,0 g	150 g	Boost Glider, Rocket Car
0360	C6-0	10,0 Ns	6 N	ca. 1,6 s	0 s	18,7 g	8,9 g	160 g	Booster Stage
0363	C6-3	10,0 Ns	6 N	ca. 1,6 s	ca. 3 s	20,5 g	9,5 g	160 g	Single Stage, Final Stage
0365	C6-5	10,0 Ns	6 N	ca. 1,6 s	ca. 5 s	21,2 g	10,2 g	150 g	Single Stage, Final Stage
0367	C6-7	10,0 Ns	6 N	ca. 1,6 s	ca. 7 s	22,0 g	11,0 g	140 g	Single Stage, Final Stage
0369	C6-P	10,0 Ns	6 N	ca. 1,6 s	verschlossen / plugged	19,9 g	8,4 g	160 g	Electronic recovery, Cluster
0430	D3-0	19,0 Ns	3 N	ca. 6,0 s	0 s	28,0 g	17,2 g	200 g	Boost Glider, Rocket Car
0439	D3-P	19,0 Ns	3 N	ca. 6,0 s	verschlossen / plugged	27,7 g	16,7 g	200 g	Boost Glider, Rocket Car
0490	D9-0	20,0 Ns	9 N	ca. 2,1 s	0 s	26,3 g	15,5 g	300 g	Booster Stage
0493	D9-3	20,0 Ns	9 N	ca. 2,1 s	ca. 3 s	27,1 g	16,1 g	300 g	Single Stage, Final Stage
0495	D9-5	20,0 Ns	9 N	ca. 2,1 s	ca. 5 s	27,6 g	16,6 g	250 g	Single Stage, Final Stage
0497	D9-7	20,0 Ns	9 N	ca. 2,1 s	ca. 7 s	28,2 g	17,2 g	200 g	Single Stage, Final Stage
0499	D9-P	20,0 Ns	9 N	ca. 2,1 s	verschlossen / plugged	26,5 g	15,0 g	300 g	Electronic recovery, Cluster

Verwendung / Usage

- Einstufige Raketen / Single Stage
- Letzter in Mehrstufenraketen / Final Stage in multi-stage rocket
- Mehrstufen Treibsatz / Booster Stage
- Raketengleiter / Boost Glider
- Raketenauto / Rocket Car
- Raketen mit elektronischer Bergungssystemauslösung / Rockets with electronic recovery systems
- Bündelung / Cluster

Die Motoren werden in Packungen zu 6 Stück mit beiliegenenden Zündschnüren geliefert. Alternativ sind Elektroanzünder erhältlich. Die Motorlänge beträgt 70 mm und der Durchmesser ist 17,8 mm.
Each motor type comes in a pack of six, including safety fuses for igniting. Electric igniters are separately available. Motor dimensions are 17,8 mm in diameter and 70 mm in length.

1.2 Forces acting on the rocket

Before you can do orbital calculations, it is important to make a good assessment of how forces act on the rocket. In this calculation exercise we will use the following three forces; thrust from the engine, gravity and air resistance.

The reason the forces that act on the rocket are so important is because they make it possible to calculate acceleration, speed and position of the rocket using Newton's 2. Law.

Newtons 2. law $$\Sigma F = ma \qquad (5)$$

If the acceleration is known, the movement of the rocket can be calculated using the motion equations

Motion equation 1 (Speed) $$\frac{dv}{dt} = a(t)$$

assuming constant acceleration $$v(t) = \int a\,dt = at + v_0 \qquad (6)$$

Motion equation 2 (Position) $$\frac{dr}{dt} = v(t)$$

assuming constant acceleration

$$r(t) = \int v(t)\, dt = \int (at + v_0)\, dt = \frac{1}{2}at^2 + v_0 t + r_0 \qquad (7)$$

Another way to find the speed of a rocket is to use the preservation of the amount of motion in a closed system.

Preservation of momentum $$\frac{d}{dt}(p_0 + p_1) = 0 \qquad (8)$$

$$\frac{d}{dt}(m_0 v_0 + m_1 v_1) = 0$$

Konstantin Tsiolkovsky derived what is known recently as the rocket equation by utilizing the preservation of momentum. We can now use this equation to calculate the ideal end speed (v_i) of the vacuum rocket, without the influence of gravity. In the equation, enter the starting mass of the m_0, while m_1 indicates the mass at burnout. The application of this equation is limited since it assumes an immediate speed change and no influence of external forces.

The rocket equation $$\Delta V = v_e \ln\left(\frac{m_0}{m_1}\right) \qquad (9)$$

Rockets sent up from the earth's surface are heavily influenced by air resistance and turbulence. The air resistance is the force of the gas particles in the air that counteracts the rocket's movement. Therefore, the air resistance always works in the opposite direction of the movement of the rocket. To model the influence of air resistance on model rockets, we use formula 10.

Air resistance/drag $$F_{drag} = \beta v^2 \qquad (10)$$

The value β we call air resistance factor. This can be calculated if one knows air resistance coefficient (C_d – see Table 4), air density (ρ) and rocket's largest frontal area of the rocket (A_{max}).

Air resistance factor $$\beta = \frac{1}{2} C_d \rho A_{max} \qquad (11)$$

1.3 Stability

Stability is the property rockets must stay in a certain direction and oppose directional changes. An unstable rocket is therefore unsafe to send up because it is impossible to predict where it will end up.

To determine the stability of the rocket, we use two characteristics of the rocket. "Center of Gravity" (C_G) and "Center of Pressure" (C_P). For the rocket to be stable, the location of the C_G must be in front of the C_P on the rocket. Before you can decide this, C_G and C_P must be known. C_G is the same as the center of gravity and is relatively easy to find by determining the balancing point of the rocket. It's in C_G that gravity acts around. It is also the point of rotation of the rocket. C_P is the point where the aerodynamic forces on the rocket are in balance. This point can be found using the "Cardboard cutout" method and done as follows:

1. Make an accurate outline of the rocket on a cardboard board.
2. Cut out the outline.
3. Find the center of gravity on the cut-out outline. This point will then be the C_P for your model rocket.
4. Measure the length from the tip to C_P on your cardboard cut-out. This will then be the length (l_{cg}) from the nose tip to C_G on your model rocket.

To determine whether the rocket is stable, one can use stability margin (SM). If the stability margin is less than 1, the rocket is in theory unstable and should be adjusted. If the margin of stability is greater than 2, the rocket is over-stable. That is, it is a little heavy in the front and the rocket will often turn into the wind. This is not critical for the launch.

Stability Margin $$\mathrm{SM} = \frac{l_{cg} - l_{cp}}{d_r} \tag{12}$$

- l_{cg} - Length measured from the nose cone to C_G
- l_{cp} – Length measured from the nose cone to C_P
- d_r – Maximum tube diameter of the rocket

Figure 2. The margin of stability must be between 1 and 2 for the rocket to be stable

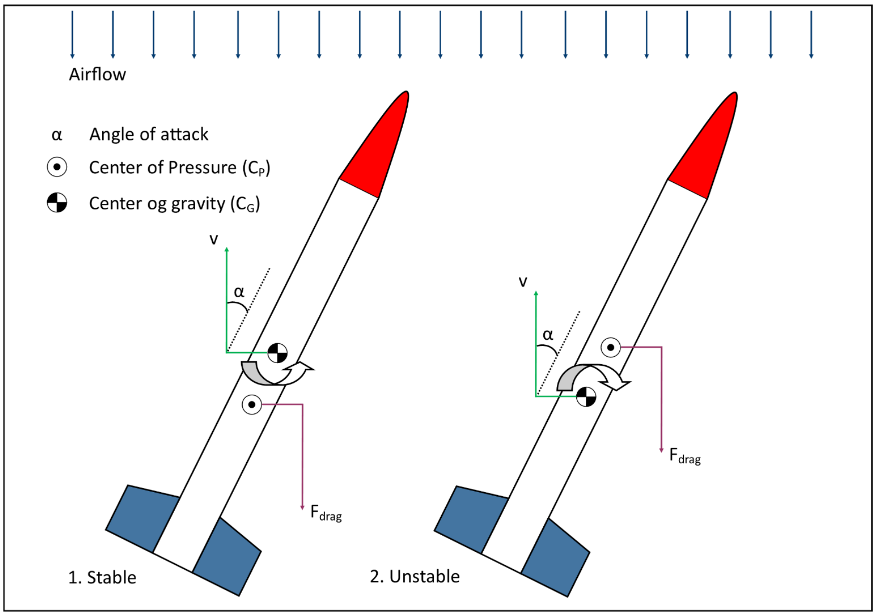


Figure 3. Air resistance always works in the opposite direction of speed. If CP is placed in front of CG such as in case 2, this causes the rocket to become unstable.

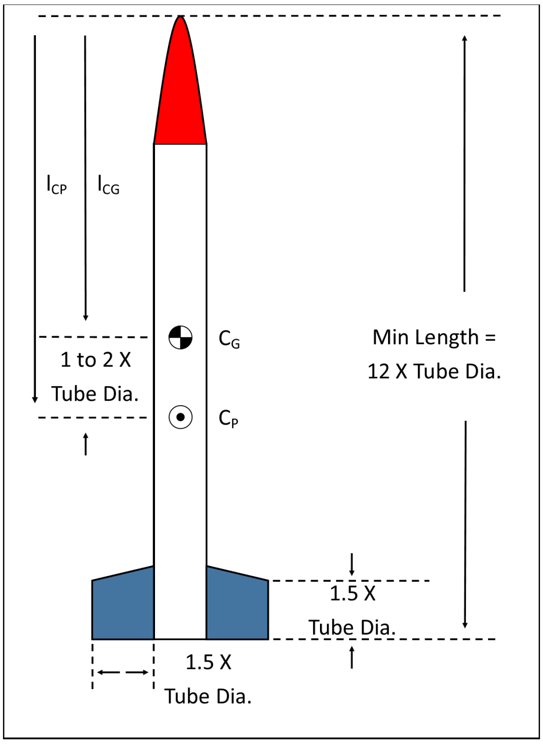


Figure 4. Guidelines for the rocket to achieve stability (Source: Model Rocket – T.S. Van Milligan)

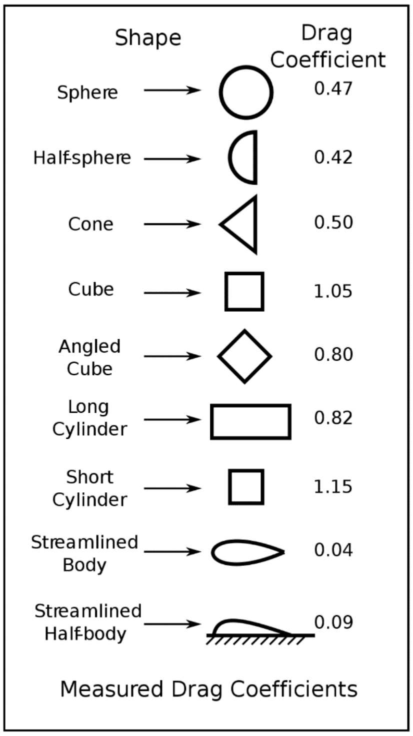


Table 4. The value of different air resistance depending on the shape.
(Source: Wikipedia – Drag coefficient)

2 Calculations

2.1 Technical specifications for rocket motor

a) Find the rocket motor type (e.g. B4-0) in the Table 3 and note down propellant mass (m_p) and maximum lifting weight (m_{max}). Is the total weight of the rocket within the maximum lifting weight?
b) Find the engine's total impulse (I_t), burn time (t_b) and delay charge (t_d).
c) Calculate the average thrust (F_{ave}) and average fuel consumption (q_p).
d) Calculate the motor exhaust speed (v_e) and the specific impulse (I_{sp}).

2.2 Model rocket orbital calculations

a) Assume that the rocket is in space without the influence of air resistance or gravity. Use Newton's 2 law and the motion equations to calculate the ideal end speed (v_i) after the rocket engine has burned out. Remember that the mass of the rocket changes when the rocket engine burns. Use average mass for the rocket to calculate the end speed.
b) Assume that the rocket is fired straight up from the Earth's surface (elevation 90°). Calculate the speed at burnout (v_{max}) without air resistance.
c) Calculate the burnout altitude (h_b) based on the response from the previous task.
d) Calculate the time it takes from burnout to the rocket reaches apogee (h_{max}). This time is called the free flight time (t_f). How does the free flight time correspond to the delay charge?
e) Calculate the maximum altitude/apogee (h_{max})

2.3 Track calculations for the model rocket with elevation

a) Assume that the rocket is fired from the Earth's surface with a 75°. We're still disregarding air resistance. What will the speed at burnout (v_{max}) be in x- and y-direction be in this case?
b) Calculate apogee (h_{max}) in this case.

2.4 Aerodynamics

a) Calculate the rocket's largest front area (A_{max}). Remember to include the front area of the fins.
b) Calculate the air resistance factor (β). Use $\rho = 1.25$ kg/m^3 and Table 4 to estimate C_d.
c) Plot air resistance (F_{drag}) on the next page for these speeds:

0	$0{,}25v_{max}$	$0.5v_{max}$	$0.75v_{max}$	v_{max}

d) Is the result likely? Comment.

e) Draw a sketch of the rocket with the forces that work on it in the following situations:
    i. When the rocket is at rest on the launch ramp
    ii. When rocket is in flight with thrust from the rocket motor
    iii. When the rocket is in free flight without thrust from the rocket motor

2.5 Stability

a) Select C_G on the cardboard. Cut out the outline of the rocket and find C_P using the "Cardboard cutout" method. Select C_P on the cardboard.
b) Calculate the margin of stability of the rocket. Is the rocket stable?
c) Comment on what will happen to C_G while the rocket is in the air.

2.6 The rocket equation

a) Calculate the rocket's ideal speed (v_i) using the rocket equation.

2.7 Rocket simulation using Rocksim

a) Open Rocksim
b) Load the correct design of your rocket e.g. QUEST_Courier.RKT
c) Select mass override and modify the settings so that CG fits your rocket.
d) How does the software's calculation of CP fit with your calculation? Comment.
e) Select the correct rocket motor for your rocket e.g. C6-3. (If the rocket motor is not available, motor files can be downloaded from https://www.apogeerockets.com/RockSim/Adding_Motors
f) Set initial conditions to no wind and a 75° elevation angle.
g) Run the simulation and find the speed at burnout (v_{max}) and the maximum altitude (h_{max})
h) Compare your calculations with the results from the simulations. Comment.

Plot air resistance - Fdrag